- The Adjustment for "Antenna" represents the antenna efficiency of the configuration being designed for. It shall represent the mean losses for that antenna configuration relative to a vertically polarized $\lambda/2$ dipole. For portables it shall include body absorption, polarization effects, and pattern variations for the average of a large number of potential users. For mobiles, it shall include losses for pattern variation for the mounting location on the vehicle and coaxial cable.

  Mobile type unit antenna height corrections shall also be included under this definition. The formulas from Hata [18] are to be employed (Sections 5.1.2.1 and 5.1.2.2).

- User adjustment is for specific usage as necessary for determining portable reliability when operating in a vehicle or in a building with specified penetration loss(es).

- The Acceptance Test Plan (ATP) Target for a hypothetical system is then the absolute power defined by the Static Threshold minus the difference between C_f/N - C_s/N plus the antenna adjustment and any usage adjustment required. For example if the static threshold is -116 dBm (C_s/N = 7 dB (arbitrary)), and -108 dBm is the Faded Threshold (C_f/N = 15 dB), the Fading Margin is 8 dB. This may not be enough for the specific CPC required. If the C_f/N for the desired performance level is 17 dB, then the fading margin is 10 dB, and the Faded Threshold becomes -106 dBm. If the portable antenna has a mean gain of -10 dBd and building losses of 12 dB are required then the average power for the design at street level should be 22 dB greater than -106 dBm (-84 dBm) for this example configuration. Table 5 in Appendix-A provides the *projected* CPC requirements for DAQ 3, 3.4, and 4.

  This establishes the average power which should be measured by a test receiver that has been calibrated to offset its test antenna configuration and cable losses. For example, if the design was for a portable system and the test receiver is using a $\lambda/4$ center mounted antenna with 2 dB of cable loss then a correction factor of -1 dB is applied for the antenna to reference it back to a $\lambda/2$ dipole plus an additional -2 dB for cable loss, -3 dB which would modify the pass/fail criterion from -84 dBm to -87 dBm.

- The Design Target includes the necessary margins to provide for the location variability to achieve the design reliability and a "confidence factor" so that average measured values will produce the CPC. For example, if the desired minimum probability of achieving the CPC is 90%, and a design actually produces such a condition, 50% of the tests would produce results greater than the 90% value and 50% would produce results less than the 90% value. A minor incremental increase in the design would allow the 90% design objective to be achieved. The necessary correction factor varies with the system parameters as indicated in Section 5.8.

- The final element in the prediction involves the actual propagation model, which predicts the mean loss from the transmitter site to a specific predicted location at some probability. The specific electromagnetic wave propagation model selected is critical as the system design, simulation, and modeling accuracy versus system performance will be dependent upon the validity and universality of the selected model. Section 5.0 contains the recommended models and methodology. Section 3.6.2.2 recommends when to use them. The completion of a specified ATP, where close agreement between predicted and measured values is achieved, essentially validates the specific models used. It is recommended that the specific models be employed for system coverage and for frequency reuse and interference predictions to assure consistency and long term validity.

3.6 Parametric Values

The data provided in Table 5 of Appendix-A were voluntarily provided by the manufacturers as "*projected*" values for system design and spectrum management. Publication of these data does not imply that either the manufacturers or TIA guarantees the conformance of any individual piece of equipment to the values provided. Users of these parametric values should validate these values with their supplier(s) to ensure applicability.

3.6.1 BER vs. E_b/N_o

The measurement of E_b/N_o vs. BER for both static and faded conditions is commonly made. For conventional technology implementations, this can be converted to static and faded C/N values with the following equation:

$$\frac{C}{N} = \frac{E_b}{N_o} + 10 Log \frac{[BitRate(Hz)]}{[ENBW(Hz)]} \qquad \text{[Eq. 4]}$$

The ENBW for a known receiver can be used, or a value may be selected from standard receiver bandwidths, to determine faded C/N values for various CPCs. Table 3 in Appendix-A includes the ENBW for various configurations.

From the known static sensitivity and its C_s/N, the value of N, the Thermal Noise floor can be calculated. Based on N and the requirement for $C_f/(\Sigma I+\Sigma N)$ from the faded reference sensitivity for a specified CPC, the absolute value of the average power required is known if the various values of I are also known. The coverage prediction model will predict the value of I.

For example, if E_b/N_o for the reference sensitivity is 5.4 dB for a C4FM receiver (ENBW = 5.76 kHz, IMBE vocoder) at -116 dBm then the C_s/N = 5.4 + 10 Log 9,600/5,760 = 7.6 dB. The calculated Inferred Noise Floor is then -123.6 dBm. From TSB102.CAAB the faded reference sensitivity limit is -108 dBm. This implies a C_f/N = 15.6 dB for 5% BER. If the specified CPC (DAQ = 4) requires 1% BER, then the C_f/N would be appropriately increased by its appropriate value, e.g., 15.6 dB to 21.2 dB. [These numbers are based on the specified minimum performance as listed in TSB102.CAAB clauses 3.1.4 and 3.1.5. The increase for improving 5% BER to 1% BER is from Table 5 of Appendix-A.] Thus the mean power level to provide this performance would be -123.6 + 21.2 = -102.4 dBm.

In a Noise Limited System, the C/N of -102.4 dBm would be the faded performance threshold. In an Interference Limited system, the requirement for $C/(\Sigma I+\Sigma N)$ where ΣI's is, for example, >>N, would require that the design C be 21.2 dB higher for the minimum probability required to provide the CPC at the worst case location. The computer simulations recommended can accurately predict this probability.

3.6.2 Co-Channel Rejection

Different modulation types and implementations require different co-channel protection ratios. The significance of Co-Channel Rejection goes beyond operation in co-channel interference: as measured per TIA TSB102.CAAA, Co-Channel Rejection is equivalent to the static IF carrier-to-noise ratio (C_s/N) required to obtain the sensitivity criterion of the receiver under test. Therefore, a receiver's Co-Channel Rejection number can be used to determine a receiver's IF filter noise floor. This is done using the formula:

$$\text{Noise Floor} = \text{Reference Sensitivity} - C_s/N \qquad \text{[Eq. 5]}$$

The receiver noise floor will be used in the interference model presented in the sections to follow.

Column 2 of Table 5 in Appendix-A gives Co-Channel Rejection values, i.e., static sensitivity in terms of IF carrier-to-noise ratio for the reference sensitivity listed, for many current modulation types.

3.6.2.1 Channel Performance Criterion

Criteria for channel performance are listed in Table 5 of Appendix-A.

3.6.2.2 Propagation Modeling and Simulation Reliability

For public safety agencies, it is recommended that the CPC be applied to 97% of the prescribed area of operation in the presence of noise and interference. Law enforcement and public safety systems should be designed to support the lowest effective radiated power subscriber set intended for primary usage. In most instances this will necessitate systems be designed to support handheld/portable operation. In these instances it is recommended the lowest practicable power level mobile/vehicular radio be assumed. If direct unit-to-unit communications are a primary operational modality, it is recommended that per-channel power control be used, where available, to minimize system imbalance and interference potential. Special consideration of this modality is required as unit-to-adjacent channel unit interference potential is increased.

For Land Mobile Radio (LMR) systems other than public safety, it is recommended that the CPC be applied to 90% of the prescribed area of operation in the presence of noise and interference. Non-public safety systems should be designed to support the typical effective radiated power subscriber set intended for primary usage. In most instances this will necessitate systems be designed to support mobile/vehicular operation. Handheld/portable operations are often secondary. In all instances it is recommended the lowest practicable power level mobile/vehicular radio be assumed. If direct unit-to-unit communications are a primary operational modality, it is recommended per channel power control be used, where available to minimize system imbalance and interference potential. Special consideration of this modality is required as unit-to-adjacent channel unit interference potential is increased. LMR systems that make primary use of handheld/portables are advised prohibit mobile station operation at power levels significantly greater than the design level used for handheld/portable usage.

3.6.2.3 Protected Service Area (PSA)

To determine suitability for assigning channels, a determination of whether the user can qualify for a Protected Service Area (PSA) is required. If the user does not qualify, then it is assumed that sharing will occur. The next requirement is whether the user can monitor the channel before transmitting so as to prevent interfering with current usage. An example of a simple weighted ordering process to select from candidate channels is provided later.

3.6.2.3.1 Proposed System Is PSA

1. Based on the Service Area defined and the appropriate licensing rules, limit the evaluation area to include only those interfering systems which can have a direct impact on the applicant's PSA.
2. Eliminate candidate channels with overlapping co-channel operational service areas.
3. Re-evaluate the remaining candidate channels by quickly evaluating potential signal(s) overlapping service areas using the following simplified prediction method: Use the recommended models, procedures, and ERP adjustments for Adjacent Channel Coupled Power in a "coarse" mode to reduce the number of candidate channels for later detailed evaluation.
4. From the remaining candidate channels, start by calculating the Service Area CPC Reliability of the PSA under evaluation due to noise and all interference sources (co- and adjacent channel interference from PSAs and non-PSAs) using the "fine" mode.
5. When a candidate channel has been identified as meeting the licensee's requirements, an evaluation of the incumbent channels due to the applicant should be made to determine the interference impact to incumbents.

6. If Step 5 produces a successful assignment, the process is complete. Alternatively, it can be continued to evaluate the remaining candidate channels, looking for an optimal solution. It is anticipated that this alternative solution may involve higher fees due to the greater time and resources required.

3.6.2.3.2 Proposed System Is Not PSA

In this scenario, adjacent channels are assumed to not be capable of being monitored before transmitting. Co-channels may be monitored if they use similar type modulation.

The assignment of a non-PSA frequency assumes that, at some time, sharing will occur. Therefore, there is no optimal solution, and any immediate solution may change in the future. Numerous tradeoffs and coordinator judgment will be required out of necessity. For that reason, this section will identify some of the factors that could potentially rank candidate channels for a recommendation. Weighting factors and the way they are applied are not specified. A similar coverage evaluation process as defined in Section 3.6.2.3.1, in conjunction with the judgmental factors, should be applied.

1. Based on the Service Area defined and the appropriate licensing rules, limit the evaluation area to include only those interfering systems which can have a direct impact on the applicant's Service Area.
2. Eliminate candidate channels using the following judgmental factors:
   - Number of licensees
   - Simplex base-to-base interference potential, point-to-point path
   - Number of units shown for each incumbent
   - Overlap of service areas
   - Similar size of co-channel service areas
   - Potential for adjacent channel interference due to overlapping service areas, potential of the near/far problem
   - Potential for adjacent channel interference due to signals overlapping service areas
   - Common or nearby site compatibility
   - Time of day utilization
   - Competition, same type of business
   - Ability to monitor before transmitting
   - Compatibility of modulation to allow monitoring of "over the air audio"
   - Use of encryption
   - Use of trunking
     - ◆ Dedicated control channel
     - ◆ Non-dedicated control channel
3. Re-evaluate the remaining candidate channels by quickly evaluating potential signal(s) overlapping service areas using a simplified prediction method. This method should use the recommended models, procedures, and ERP adjustments for Adjacent Channel Coupled Power in a "coarse" mode to reduce the number of candidate channels for later detailed evaluation..

4. From the remaining candidate channels, start by calculating the Service Area CPC Reliability of the non-PSA under evaluation due to noise and all interference sources (co- and adjacent channel interference from PSAs and non-PSAs).
5. When a candidate channel has been identified as meeting the licensee's requirements, an evaluation of the incumbent channels due to the applicant should be made to determine the interference impact to incumbents.
6. The judgmental factors of Step 2 should be re-examined for applicability.
7. If Step 5 produces a successful assignment, the process is complete. Alternatively, the process can be continued to evaluate the remaining candidate channels, looking for an optimal solution. It is anticipated that this alternative solution may involve higher fees due to the greater time and resources required.

3.6.2.3.3 Example of Ordering

Consider a case with four successful candidates. Each has two co-channel PSAs and three have adjacent channel PSAs. Refer to Table 6 in Appendix-A for the example.

3.6.3 Interference Prediction

It is assumed that for any modulation combination, it is valid to treat adjacent channel interference as additional noise power that enters a receiver's IF filter. Interference between different modulation types may be calculated based on the power spectrum of the given transmitter modulation and the IF filter selectivity and IF carrier-to-noise ratio required to obtain the specified CPC in a Rayleigh faded channel. The $C_f/(I+N)$ then becomes a predictor of CPC.

The $C_f/(I+N)$ required for the "victim" system to meet its required CPC must be known in order to determine an interference level. The subscript "f" indicates that the carrier-to-noise ratio is determined for Rayleigh *faded* conditions. When performing interference calculations, it is important to use faded carrier-to-noise values since faded conditions more accurately represent the field environment.

Columns 3-5 of Table 5 in Appendix-A list projected CPC requirements for mainstream modulation techniques at various DAQ levels in faded conditions. For digital modulations, bit error rates associated with each CPC are given. These may be used to determine if a given $C_f/(I+N)$ exists in an actual field test application. Static reference sensitivity (C_s/N) also is given. This value can be used to determine the receiver noise floor for interference modeling. A particular manufacturer's implementation may vary from these values somewhat, but the variation is expected to be small.

A key factor in determining adjacent channel interference is the IF selectivity of the victim receiver. There is potentially wide variation in IF selectivity between manufacturers, but definition of a standard IF selectivity is helpful in defining a reproducible test. A set of prototype IF filters is given in Table 3. The filter implementations used here were selected for their ability to compactly define an explicit and reasonable implementation, not to suggest an optimum implementation for a given modulation type. Formulas also are provided for use in simulations in Table 4 of Appendix-A.

Receiver local oscillator noise also is a factor in interference. Since this is a function of receiver design, and performance may vary greatly between various implementations, and since the type of interference does not affect co- or adjacent channel performance, this factor will not be considered in the analysis. It is understood, however, that a certain noise floor due to local oscillator noise will exist.

Transmitter spectra will be modeled using measured spectrum power densities (SPDs). The SPDs are measured according to the procedures given in Section 6.6. Some are represented in tabular form in Appendix-C. The SPDs in Appendix-C are given in terms of Watts and normalized to a total transmit power of 1 Watt.

4.0 Noise

4.1 Environmental RF Noise

To determine effective receiver sensitivity, it is essential that the level of environmental noise be known. It should first be pointed out that it is seldom necessary to measure environmental noise in a mobile environment at frequencies higher than 400 MHz because it is rare for the total environmental noise to exceed kT_0b. A major exception to the foregoing statement is frequencies near 821 MHz in which the mobile can experience noise generated by non-wireline cell sites. The foregoing advice is summarized in Table 9.

4.2 Historical RF Noise Data

Noise measurements have been conducted by many researchers. One representative noise survey was that of Spaulding and Disney [9]. Their work resulted in the following RF noise equation:

$$N_r = 52 - 29.5 \log_{10} f_{MHz} \quad \text{dB} \qquad (\text{Relative to } kT_0b) \qquad [\text{Eq. } 6]$$

Where N_r is the "quiet rural" noise level relative to kT_0b. They also arrived at the following corrections for environments other than "quiet rural" should be added to N_r:

Rural: 15 dB   Residential: 18 dB   Business: 25 dB

The total cannot be less than 0 dB (relative to kT_0b).

Environmental noise is highly variable even within the same environment and the only certain means of determining the level of environmental noise (and thus the effective sensitivity) is to conduct a noise measurement program.

4.3 RF Noise Measurement Methodology

4.3.1 Receiver Selection

By far, the best tool for making a noise measurement is a receiver designed specifically for that purpose, such as an Rohde & Schwartz RSVS (example only). This type of receiver has numerous advantages, and two disadvantages when compared to a communications receiver:

- A specialized measurement receiver is expensive.
- The measurement bandwidth is somewhat inflexible.

This last may not be much of a disadvantage, since the noise spectral power density can easily be calculated and the noise power in any given ENBW can be calculated from that.

A communications receiver can also be used for making noise measurements. Although they do not have the many features provided by a measuring receiver, they are adequate for the job when properly applied and do have a small number of advantages over measuring receivers, including low cost and having the exact bandwidth that is needed for the given application.

If a communications receiver is to be used, consideration should be given to adding a low noise preamplifier to increase the measurable range at the low end. Otherwise, noise that is below the measurement threshold but may still contribute to degradation will be ignored. Care should be exercised such that intermodulation products can be produced, distorting the measurements.

4.3.2 Antenna Selection

Since noise originates from all directions, an argument can be made for measuring noise by using an antenna that is sensitive in all directions; i.e., one with an isotropic pattern. In the real world, however, specific types of antennas are used in land mobile communications and they typically have a great deal of vertical directivity. To match the results to the hardware that a user will be using in the real world, the measurements should be taken with the type of antenna that will be used by the typical user.

Radio frequency noise is frequently expressed in terms of dB above the noise floor (kT_0b) or in terms of spectral power density (in units such as dBm/kHz). Using such terms rather than the received signal level has the advantage of making the measurement "portable" to receivers with any noise bandwidth. To do so, of course, it is necessary to know the following in addition to the received signal level: (a) the gain or loss of the antenna system (including cable and connector losses), and (b) the measuring receiver's ENBW.

4.3.3 RF Noise Measurement in a Mobile Environment

A typical receiver's sensitivity can be stated in terms of a carrier to noise value; e.g., a particular receiver may require a 7 dB C_s/N to produce the static reference sensitivity. Knowing the noise power at the frequency of interest at a given location and the values from Table 5 of Appendix-A allows, the user to calculate the receiver's sensitivity for the desired CPC in that environment.

A standard communications receiver can be used for the noise measurement. If the receiver's Received Signal Strength Indicator (RSSI) bus is considerably more sensitive than the sensitivity corresponding to the desired CPC, a preamplifier will not be necessary to extend the measurable range; otherwise, a low noise preamplifier must be connected between the antenna and the receiver. The receiver must then be precalibrated. Connect a signal generator to the input of the preamplifier (or the receiver if no preamplifier is used). In the low signal range, this calibration should be done in 1 decibel intervals. Each calibration point should be repeated many (≥ 30) times to ensure a valid reading. All of this may be automated by a data acquisition device/system.

The actual readings are taken by driving around the evaluation area using a test setup to take readings in an automated fashion. A typical test setup would consist of the antenna and receiver, a notebook computer, and an analog-to-digital (A/D) converter on a PCMCIA card. A more fully automated system could include Global Positioning Satellite (GPS) or Differential Global Positioning Satellite (DGPS) data to eliminate user interface for location information.

A computer program can be written to take the necessary readings subtract the effects of the antenna system, compare the results to the calibration curve, and note the results corresponding to a given location. This will give a noise power value, typically in dBm. To arrive at the noise level relative to kT_0b, one must know the Equivalent noise bandwidth. Knowing that, one merely subtracts kT_0b from the (already determined) noise power.[1]

After taking the data, the user can then establish noise contours for the area of interest. Using this information, it is possible to, knowing the receiver's C_s/N performance for a given CPC, establish the receiver's effective sensitivity on a geographic basis.

4.3.4 Fixed RF Noise Measurement

An entirely different approach is taken to doing site noise measurements. Connect a coaxial switch so that one pole is connected to a simulation of the proposed antenna system, and the other pole is connected to a matched coaxial load. The moving contact is connected via an isolated RF coupler (such as a directional coupler) to a receiver similar to the one that will be used in the proposed system. Switch the coaxial switch so that the load

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[1] For ease of calculation, it should be noted that the value of kT_0 is -144 dBm. Note that to use this number, bandwidth must be expressed in kHz.

is connected in. Connect a (1 kHz 60% system deviation) modulated RF signal generator to the isolated port of the coupler. Increase the RF level of the RF signal generator until the SINAD and/or BER produced by the receiver approaches the value that corresponds to the desired CPC. Note the RF level. Next, switch the coaxial switch to the antenna system. Increase the RF level until the SINAD reading again reaches the desired level. Note the RF level. The difference in levels is the amount by which the specified sensitivity must be increased to arrive at the effective sensitivity. It should be noted that it is very advisable to make this measurement at several times throughout the workday to account for variations in the use of the RF sources on the site.

The method discussed in the previous paragraph is identical to that discussed in Section 6.8.5.1. See that section for a more detailed discussion.

The noise power can be ascertained from this measurement by knowing the required C_s/N for the target CPC. (See Section 3.6.2. and Table 5 of Appendix-A) Using the (previously calculated) effective sensitivity and subtracting out the required C_s/N, yields the received noise power. Knowing the receiver's ENBW, it is a simple matter to calculate the noise relative to kT_0b merely by subtracting kT_0b (in dB units) from the received noise power (in dB units).

4.4 Symbolic RF Noise Modeling and Simulation Methodology

4.4.1 Receiver/Multicoupler Interference

Receiver intermodulation effects are rarely considered in system interference. When tower mounted amplifiers and/or amplified receiver multicouplers are used they can dramatically increase the link margins, but introduce intermodulation which is detrimental.

The amount of gain provided has a direct impact on the overall noise figure of the cascaded combination of elements and on the intermodulation performance. As linear systems come into existence an increased awareness of the tradeoffs is necessary to more accurately calculate the effect. Adding gain without determining its overall effect. on the system performance and interference potential should not be tolerated.

Some base stations specify the performance sensitivity at the input to the receiver multicoupler. Most base stations receiver noise figures fall between 9 and 12 dB, with a typical design noise figure of 10 dB. The overall receiver multicoupler scheme has a composite noise figure of between 5 and 7 dB, with 6 dB being a typical design value. With a true noise figure of 4 dB, 25 dB of gain, followed by 16 dB of splitting loss and one dB of cable loss, the resulting noise figure of the cascaded chain can be calculated using the formula:

$$NF_c = NF_1 + [NF_2 - 1]/G_1 + [NF_3 - 1]/[G_1 \bullet G_2] \qquad \text{[Eq. 7]}$$

where:

NF is the Noise Factor (numeric)
G is the Gain of an Amplifier (numeric)

$NF_1 = 4.0\text{ dB} = \mathbf{2.5}.$ $\qquad G_1 = 25\text{ dB} = \mathbf{316}$
$NF_2 = 17\text{ dB} = \mathbf{50}$ $\qquad G_2 = -17\text{ dB} = \mathbf{0.02}$
$NF_3 = 10\text{ dB} = \mathbf{10}$
$NF_c = 2.5 + [50 - 1]/316 + [10 - 1/[316 \bullet 0.02] = 4.08 = 6.1\text{ dB}$

From this example, the overall noise figure of the combination is improved over the base station receiver by itself but degraded from the noise figure of the multicoupler's amplifier. By increasing the gain of the amplifier, and reducing the loss in the splitter, the cascaded noise figure trends toward the noise figure of the multicoupler. However, all the excess gain tends to increase the level of intermodulation products for components down stream. With linear systems, a specification that limits the amount of "excess gain" that can be introduced prior to the base receiver may be necessary to keep the entire system operating within a linear region.

To determine the absolute power level of the intermodulation products requires the use of the Third Order Intercept point (IP^3). Considerable confusion exists around the IP^3 due to manufacturers specmanship. Most manufacturers use the Output Third Order Intercept Point (OIP^3) as it produces a higher number. Reducing the manufacturers OIP^3 by the gain of the amplifier calculates the Input Third Order Intercept Point (IIP^3). This is more useful as one can now determine the intermodulation products with respect to the desired carrier and design noise threshold, adjusting absolute levels by selecting gain and loss elements.

4.4.2 Intermodulation

A receiver with an 80 dB Intermodulation Rejection (IMR) has an IIP^3 in the 0 to +5 dBm range. To measure the IMR, start with the static sensitivity criterion, such as 12 dB SINAD, C_s/N = 5 dB for an analog FM radio with 25 kHz channel spacing. The desired is increased by 3 dB and two interfering signals are injected. One is the adjacent channel and the other is the alternate channel. In this case, 2 times the adjacent channel, minus the alternate channel will create a product that falls back on the same frequency as the desired. The two signals are increased at the same level until the 12 dB SINAD performance specification is again reached. The difference between the intermodulating signals and the original reference is the IMR of the receiver.

In Figure 3, if the IMR specification is 80 dB, and the 12 dB SINAD is -119 dBm, (0.25 µV), the following test would be conducted. Inject -119 dBm and measure 12 dB SINAD.

The inferred design noise threshold would be -124 dBm. Increase the desired signal level to -116 dBm, 3 dB boost. Inject the adjacent and alternate channels, increasing them until 12 dB SINAD is once again obtained. With a receiver of 80 dB IMR, the adjacent and alternate channels should be 80 dB above the 12 dBS, -39 dBm. This once again produces a C_s/N of 5 dB, 12 dBS, comprised of the -124 dBm design thermal noise and another -124 dBm noise equivalent from the interference from the IMR. The combined noise sources equal -121 dBm versus the desired signal at -116 dBm. Figure 3 illustrates a graphical solution for the IIP^3 of +3.5 dBm. Two slopes are constructed. A 1:1 relationship from the design noise threshold and a 3:1 slope for the third order products offset by (80 + 5) 85 dB at the design noise threshold. A formula for this relationship is:

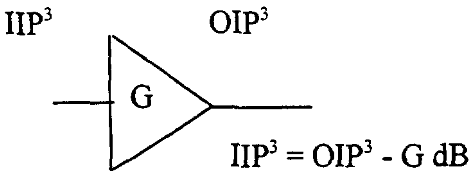
$$IMR = 2/3\ (IIP^3 - Sens) - 1/3\ (C/N\ @\ Sens) \qquad [Eq.\ 8]$$

In this example, sensitivity for 12 dB SINAD was -119 dBm with a C/N of 5 dB. If the IMR is 80 dB, the IIP^3 is = +3.5 dBm.

The preceding calculation was for a single receiver. When cascaded with a receiver multicoupler the process becomes more complex. The IIP^3 of the receiver must be found to determine the interaction with the parameters of the receiver multicoupler chain.

Receiver multicoupler manufacturers typically use the OIP^3 for their specification. Knowing the gain of the amplifier and the splitting losses one can calculate the impact on the desired and undesired portions. This will also highlight the case of when there are two amplifiers in the multicoupler chain and the gain inserted to lower the cascaded effective noise figure reduces IMR performance too much. Tower top amplifiers normally involve three amplifiers, the tower top amp, a distribution amplifier and the actual receiver.

An example will illustrate the issues. Consider the previously described base station configuration with a receiver multicoupler. The parameters and lineup are shown in Figure 4. The noise figure is calculated to be 9.2 dB, based on 12 dBS = -119 dBm, C/N = 5 dB and the ENBW = 12 kHz.



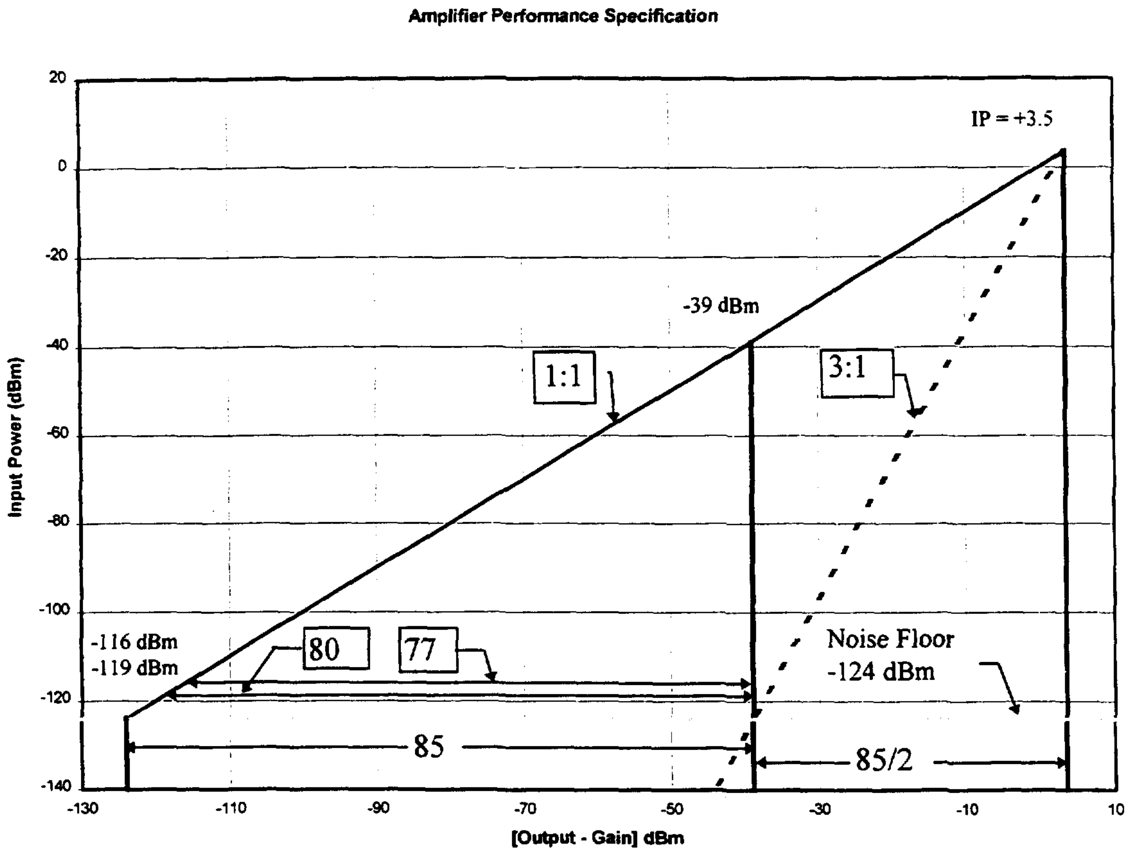
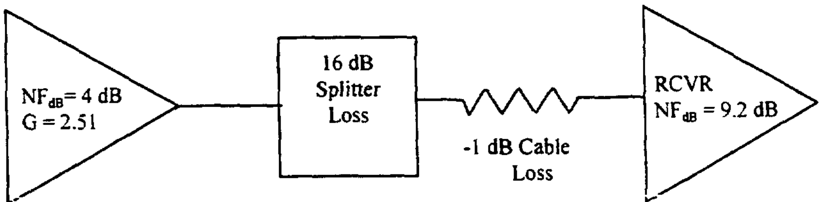


Figure 3. Amplifier Performance Specifications

The receiver multicoupler has 25 dB of gain and 17 dB of losses prior to the receiver's antenna port. The OIP^3 is given as +34 dBm. By subtracting the gain we calculate an IIP^3 of +9 dBm.



$NF_1 = 4 \text{ dB} = 2.51$ $\quad$ $G1 = 25 \text{ dB} = 316$

$NF_2 = 17 \text{ dB} = 50$ $\quad$ $G2 = -17 \text{ dB} = 0.02$

$NF_3 = 9.2 \text{ dB} = 8.32$

$$NFc = 2.51 + \frac{(50 - 1)}{316} + \frac{(8.32 - 1)}{(316)(0.02)}$$

$$NFc = 2.51 + 0.16 + 1.16 = 3.83 = 5.83 \text{ dB}$$

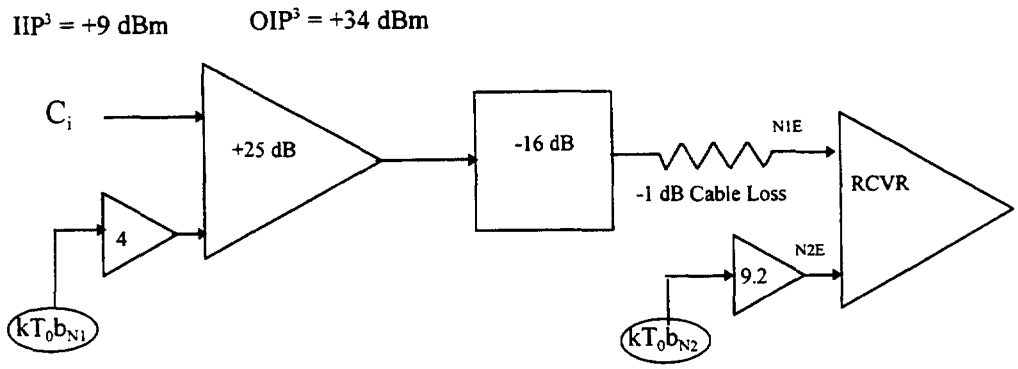
$$NF_{imp} = 9.2 - 5.83 = \underline{3.37 \text{ dB}}$$

Figure 4. Noise Figure Calculation

The traditional cascaded noise figure approach calculates an effective noise figure at the input of the multicoupler of 5.83 dB, indicating a 3.37 dB improvement in the noise figure for the combination.

4.4.3 The Symbolic Method

Symbolically all active devices are shown, in Figure 5, as a single amplifier with some known amount of gain. Inputs to the amplifier include another amplifier which has the gain of the device's noise figure which is fed from a noise source equal to the kTb value of the actual receiver. Following the flow from the first amplifier, the noise source is amplified and attenuated until it arrives at the input of the final receiver. In this case the accumulated noise power is -121.2 dBm. The receiver has its own noise source which is -124.0 dBm. The sum of these two noise sources is -119.37 dBm. To achieve a C/N of 5 dB requires that the C be -114.37 dBm. To achieve that power with the gain and losses would require a -122.37 dBm signal at the input to the first amplifier. The receivers sensitivity by itself for a C/N of 5 dB is -119 dBm so the improvement of the combination is -119 - (-122.37) = 3.37 dB, the same as calculated by the cascaded noise figure formula.



N1E = -133.2
+4.0
+25.0
-17.0
-121.2 dBm

N2E = -133.2
+9.2
-124.0 dBm

IIP^3 = +3.5 dBm
12 dBS = -119 dBm
ENBW = 12 kHz
C_s/N @ 12 dBS = 5 dB
NF_{dB} = 9.2 dB
kT_0b = -133.2 dBm

Σ(N1E + N2E) = -119.37 dBm
Ci = [-119.37 + 5] -25 +16 +1 = -122.37 dBm
Performance Improvement = -119.0 - (-122.37) = **3.37 dB**

Figure 5. Symbolic Method

This approach allows evaluating the effect of system IMR noise power. Equations 11 and 12 can be used to calculate either a relative or absolute power level for the third order product. First an equivalent signal power level must be calculated to use in this evaluation. For the classic IMR case as measured by the EIA, the equivalent signal power C_i, is:

$$C_i = \frac{2(\text{Adjacent Channel Power}) + \text{Alternate Channel Power}}{3} \qquad \text{[Eq. 9]}$$

For the EIA test, both the adjacent and alternate channels are held at the same power level. However in the field, users frequently must deal with IMR where the frequency relationships aren't that close and are unequal in power. In these cases the equivalent power to use for C_i would be to consider only the worst case which would be where the two signals have different average powers. It is also assumed that the mixer remains constant and that no additional selectivity is available. In this case:

$$C_i = \frac{2(\text{Highest Channel Power}) + \text{Lowest Channel Power}}{3} \qquad \text{[Eq. 10]}$$

An application with specific frequencies, calculates the interfering carrier levels and the intermodulation power that will result for a specific design or problem evaluation.

At the input of an amplifier:

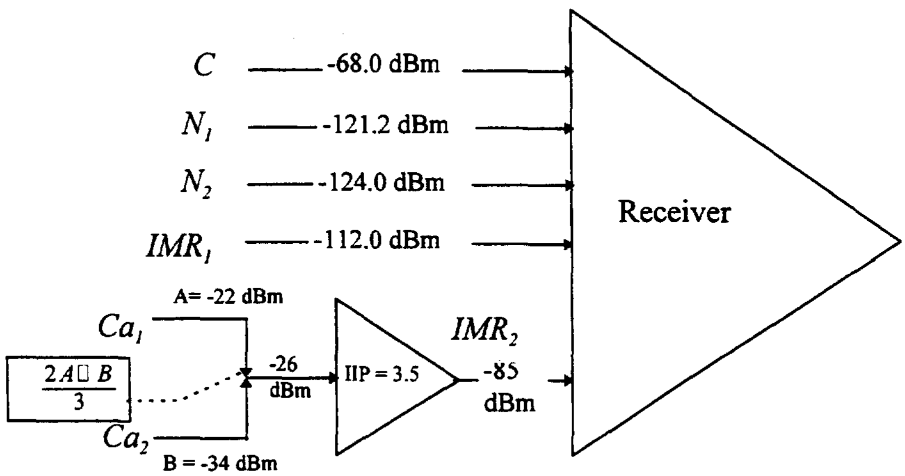
Relative IMR = 2 (IIP^3 - C_i), where C_i = Equivalent interferer. [Eq. 11]

Absolute IM Level = C_i - Relative IMR. [Eq. 12]

In most cases system designers will be interested in the level of the IM and will then follow it through the chain of amplifiers and loss elements until it arrives at the input of the last amplifier stage. At the final stage, the individual carriers also will be present and will once again produce IMR. The total noise would then be the sum of the individual noise sources and the individual IMRs, C/Σ (N + IMR). Continuing with the example, consider the following case.

The Adjacent channel power, C_{a1}, at the input to our multicoupler amplifier is -30 dBm, and the Alternate channel, C_{a2}, is -42 dBm. This is the classic 2A-B IM case. From [Eq. 10]:

C_i = [2(-30) + (-42)] / 3 = -34 dBm [Eq. 13]



$$\square Ns \square N_1 \square N_2 \square IMR_1 \square IMR_2 \square \square 85 dBm$$

$$\frac{C}{\square N \square IMR} \square 17 dB \square \; C \square \square 68 dBm$$

Figure 6. Multicoupler IMR Performance Example

The IIP^3 of the first amplifier is +9 dBm. The absolute IMR at the input of the receiver is calculated to be -34 dBm -2(43) + 25 -17 = -112 dBm. The individual C_{a1} and C_{a2} would be amplified (25 - 17) = 8 dB to -22 dBm and -34 dBm respectively. Their C_i is now -26 dBm. Thus the absolute IMR , using equations 10 and 11, introduced by the receiver itself is -85 dBm. Now there are five different inputs to the final receiver that impact its performance; the desired C, the four noise sources that must be overcome, $N_1 + N_2 + IMR_1 + IMR_2$. In this example, the IMR due to the high adjacent and alternate channels are controlling. To achieve a desired C = 17 dB above the composite noise generators requires that the signal at the input of the receiver would have to be -68 dBm to achieve our CPC for 25 kHz analog FM performance of DAQ = 3. As shown from this example, additional amplifiers in the "gain chain" can amplify high interfering signals to such a high level that IMR in unavoidable. Proper addition of attenuators is necessary to optimize the sensitivity verses IMR performance.

It is important to remember that there is a probability consideration that has to be included, and that the type of interference must also be considered. For example, if the interfering adjacent channel had the same CTCSS code, a receiver would open whenever the interference was present and no desired carrier was present. This would dramatically impact the users perception of the amount of interference.

4.4.4 Non-Coherent Power Addition Discussion

When adding powers, the values must be in some form of watts before they are added. In microwave systems the picowatt is commonly used. To add the powers, it is not necessary to convert them to a specific watt level, milliwatts, microwatts, or picowatts. As long as they all are at the same pseudowatt level they can be added and converted back and forth to the nonlinear form of decibels.

The following simple method may be used to combine powers in the decibel form. It only requires taking the dB difference of two powers and looking up in Figure 7 or Table 10 of Appendix-A a value to add to the higher power. For example, if a -113 dBm and -108 dBm are to be combined, the difference is 5 dB which from Table 10 indicates that +1.2 dB must be added to the -108 dBm for a composite -106.8. For cases with more than two power levels, the process can be repeated multiple times. P1 and P2 can be combined to Pc which can then be combined with P3 for the average power of all three.

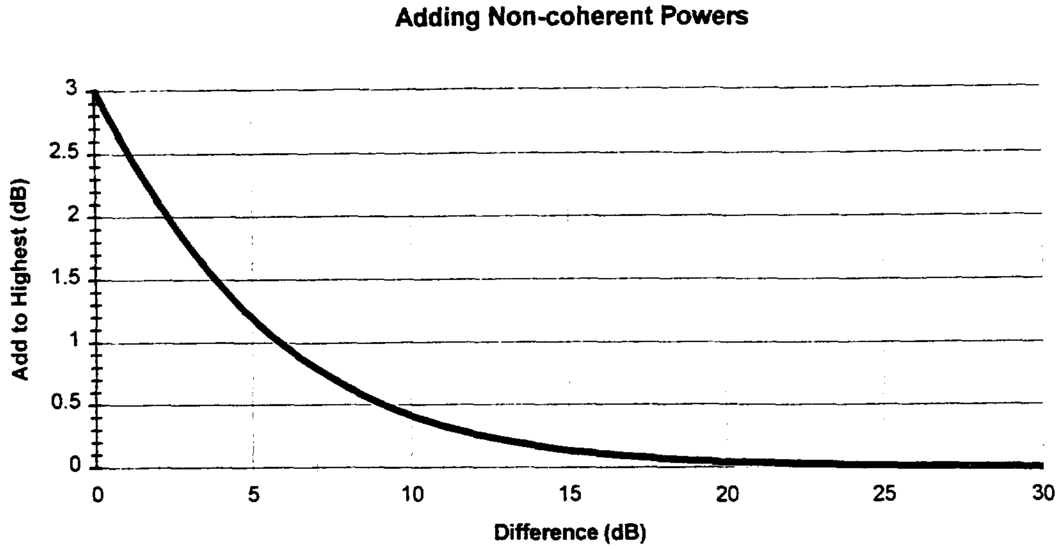


Figure 7. Adding Non-Coherent Powers

5.0 Electromagnetic Wave Propagation Prediction Standard Model

For studies involving spectrum management, two types of propagation models have been identified as appropriate. The first is a simple empirically-based model described below as the "Okumura/Hata/Davidson" model, which provides rapid calculation of path loss for line of sight conditions using terrain and land usage data.

The second model is a physical rather than empirical model, which explicitly takes into account terrain and ground clutter features present along with the great circle path from the transmitter to the receiver. It is described below as the "Anderson 2D" model. It provides more accurate path loss predictions than the "Okumura/Hata/Davidson" model under non line of sight conditions. Based on extensive comparisons with measurement data, this model produced the best overall results when compared to several other models that were evaluated. The "Anderson 2D" model is therefore recommended as the standard for frequency coordination of systems requiring a "Protected Service Area" (PSA), or other conditions where a detailed assessment of interference is desired. This process is contained in Section 3.6.2.3.1. For non-PSA systems, the rapid calculation method contained in Section 3.6.2.3.2 using the "Okumura/Hata/Davidson" model is recommended.

5.1 The OKUMURA Model

The OKUMURA model [14] is an empirical model. The results were published as curves which contain various correction factors for predicting the average power levels. When used in this section and associated subsections, the term "HAAT" refers to the HAAT in the direction of the radial under consideration, ***not*** to the overall site HAAT.

5.1.1 Hata Conversion

Hata converted the OKUMURA model for computer use [18]. He developed a series of formulas that provide OKUMURA predictions, but limited their applicability to:

- Range from Base, 1 - 20 km
- Frequency Range, 150 - 1500 MHz
- Base HAAT, 20 - 200 meters

5.1.2 Davidson Extension

Davidson has added correction factors to extend Hata's formulas back to the full range of OKUMURA and has extended the applicable distance to 300 km. This covers the following parameters:

- Frequency Range, 30 - 1500 MHz
- Base HAAT, 20 - 2500 Meters
- Range from Base, 1 to 300 km

Use the larger (greater loss) of PL or PL2 as calculated by either one of the subroutines below.

5.1.2.1 Sample OKUMURA/HATA/DAVIDSON Program - Metric

```
C SUBROUTINE TO COMPUTE THE HATA PATH LOSS FROM OKUMURA MODIFIED BY
C DAVIDSON, METRIC VERSION 2.1      10/21/96
C ASSUMES THAT THE MOBILE HEIGHT FOR MEDIUM SMALL CITY IS SUBURBAN-
C QUASI OPEN-OPEN AND FOR LARGE CITY IS URBAN
C ***************************************************************
C INPUT TO THE SUBROUTINE
C FREQ ..... FREQUENCY IN MHZ
C HEIGHT ... BASE HEIGHT ABOVE AVERAGE TERRAIN (HAAT) IN METERS
C HIMOB .... MOBILE HEIGHT IN METERS
C RANGE .... DISTANCE BETWEEN TRANSMITTER AND RECEIVER IN km
C ENVIOR ... THE ENVIRONMENT OF THE MOBILE, CHOICE OF 4
C            1) URBAN
C            2) SUBURBAN
C            3) QUASI OPEN
C            4) OPEN
C ***************************************************************
C OUTPUT OF THE SUBROUTINE
C PL .... HATA/DAVIDSON PATH LOSS BETWEEN ISOTROPIC POINT SOURCES IN DBI
C PL2 ... FREE SPACE PATH LOSS BETWEEN ISOTROPIC POINT SOURCES IN DBI
C ***************************************************************
C
      SUBROUTINE LOSS (FREQ,HEIGHT,HIMOB,RANGE,ENVIOR,PL,PL2)
      CHARACTER ENVIOR*8
C FIRST COMPUTE HATA URBAN
      PL=69.55+26.16*ALOG10(FREQ)-13.82*ALOG10(HEIGHT)+
     + (44.9-6.55*ALOG10(HEIGHT))*ALOG10(RANGE)
C SUBTRACT HATA CORRECTION FOR MOBILE HEIGHT, URBAN = LARGE CITY ELSE
```

```
C OTHER ONE.   USE 300 MHz AS THE FREQUENCY BREAK POINT.
      IF(ENVIOR.EQ.'URBAN') THEN
        IF(FREQ.GT.300) THEN
          PL=PL-(3.2*(ALOG10(11.75*HIMOB))**2-4.97)
        ELSE
          PL=PL-(8.29*(ALOG10(1.54*HIMOB))**2-1.1)
        ENDIF
      ELSE
        PL=PL-(1.1*ALOG10(FREQ)-0.7)*HIMOB+
     +   (1.56*ALOG10(FREQ)-0.8)
      ENDIF
C SUBTRACT HATA CORRECTION FOR OTHER ENVIRONMENTS
      IF (ENVIOR.EQ.'SUBURBAN') PL=PL-5.4-2*(ALOG10(FREQ/28)**2)
      IF (ENVIOR.EQ.'OPEN'.OR.ENVIOR.EQ.'QUASI O')
     +    PL=PL-40.94+18.33*ALOG10(FREQ)-4.78*(ALOG10(FREQ)**2)
      IF (ENVIOR.EQ.'QUASI O') PL=PL+5
C NOW EXTEND IT IF YOU ARE OVER THE RANGE LIMIT OR BASE HEIGHT LIMIT
       R1=20
       R2=64.38
C FOR ALL RANGES GREATER THAN 20 km ADD A FACTOR
      IF (RANGE.GT.R1) THEN
        PL=PL+(0.5+0.15*ALOG10(HEIGHT/121.92))*(RANGE-R1)*0.62137
      ENDIF
C FOR ALL RANGES GREATER THAN 64.38 km SUBTRACT A FACTOR
      IF (RANGE.GT.R2) PL=PL-0.174*(RANGE-R2)
C FOR ALL BASE HEIGHTS GREATER THAN 300 M SUBTRACT A FACTOR
      IF (HEIGHT.GT.300) THEN
        PL=PL-0.00784*ABS(ALOG10(9.98/RANGE))*(HEIGHT-300)
      ENDIF
C MAKE THE EQUATIONS THAT WORK FOR 1500 MHz GO DOWN TO 30 MHz
      PL=PL-(FREQ/250)*ALOG10(1500/FREQ)
      R3=40.238
      IF(RANGE.GT.R3) PL=PL-0.112*ALOG10(1500/FREQ)*(RANGE-R3)
C COMPUTE FREE SPACE PATH LOSS IN DBI
      PL2=32.5+20*ALOG10(FREQ)+20*ALOG10(RANGE)
      RETURN
      END
```

5.1.2.2 Sample OKUMURA/HATA/DAVIDSON Program - English

```
C SUBROUTINE TO COMPUTE THE HATA PATH LOSS FROM OKUMURA MODIFIED BY
C DAVIDSON, ENGLISH VERSION 1.2   10/21/96
C ASSUMES THAT THE MOBILE HEIGHT FOR MEDIUM SMALL CITY IS SUBURBAN-
C QUASI OPEN-OPEN AND FOR LARGE CITY IS URBAN
C ****************************************************************
C INPUT TO THE SUBROUTINE
C FREQ ..... FREQUENCY IN MHZ
C HEIGHT ... BASE HEIGHT ABOVE AVERAGE TERRAIN (HAAT) IN FEET
C HIMOB .... MOBILE HEIGHT IN FEET
C RANGE .... DISTANCE BETWEEN TRANSMITTER AND RECEIVER IN MILES
C ENVIOR ... THE ENVIRONMENT OF THE MOBILE, CHOICE OF 4
```

```
C             1) URBAN
C             2) SUBURBAN
C             3) QUASI OPEN
C             4) OPEN
C ***************************************************************
C OUTPUT OF THE SUBROUTINE
C PL .... HATA/DAVIDSON PATH LOSS BETWEEN ISOTROPIC POINT SOURCES IN DBI
C PL2 ... FREE SPACE PATH LOSS BETWEEN ISOTROPIC POINT SOURCES IN DBI
C ***************************************************************
C
      SUBROUTINE LOSS (FREQ,HEIGHT,HIMOB,RANGE,ENVIOR,PL,PL2)
      CHARACTER ENVIOR*8
C FIRST COMPUTE HATA URBAN
C EQUATIONS FROM HATA HAVE BEEN CONVERTED TO ENGLISH UNITS
      PL=86.65+26.16*ALOG10(FREQ)-15.17*ALOG10(HEIGHT)+
     + (48.28-6.55*ALOG10(HEIGHT))*ALOG10(RANGE)
C SUBTRACT HATA CORRECTION FOR MOBILE HEIGHT, URBAN = LARGE CITY ELSE
C OTHER ONE.   USE 300 MHz AS THE FREQUENCY BREAK POINT.
      IF(ENVIOR.EQ.'URBAN') THEN
        IF(FREQ.GT.300) THEN
          PL=PL-(3.2*(ALOG10(11.75*HIMOB*0.3048))**2-4.97)
        ELSE
          PL=PL-(8.29*(ALOG10(1.54*HIMOB*0.3048))**2-1.1)
        ENDIF
      ELSE
        PL=PL-(1.1*ALOG10(FREQ)-0.7)*HIMOB*0.3048+
     +   (1.56*ALOG10(FREQ)-0.8)
      ENDIF
C SUBTRACT HATA CORRECTION FOR OTHER ENVIRONMENTS
      IF (ENVIOR.EQ.'SUBURBAN') PL=PL-5.4-2*(ALOG10(FREQ/28)**2)
      IF (ENVIOR.EQ.'OPEN'.OR.ENVIOR.EQ.'QUASI O')
     +    PL=PL-40.94+18.33*ALOG10(FREQ)-4.78*(ALOG10(FREQ)**2)
      IF (ENVIOR.EQ.'QUASI O') PL=PL+5
C NOW EXTEND IT IF YOU ARE OVER THE RANGE LIMIT OR BASE HEIGHT LIMIT
      R1=12.4
      R2=40.0
C FOR ALL RANGES GREATER THAN 12.4 MILES (20 km) ADD A FACTOR
      IF (RANGE.GT.R1) THEN
        PL=PL+(0.5+0.15*ALOG10(HEIGHT/400))*(RANGE-R1)
      ENDIF
C FOR ALL RANGES GREATER THAN 40 MILES (64.38 km) SUBTRACT A FACTOR
      IF (RANGE.GT.R2) PL=PL-0.28*(RANGE-R2)
C FOR ALL BASE HEIGHTS GREATER THAN 984 FEET (300 M) SUBTRACT A FACTOR
      IF (HEIGHT.GT.984) THEN
        PL=PL-4.7*ABS(ALOG10(6.2/RANGE))*(HEIGHT-984)/1968
      ENDIF
C MAKE THE EQUATIONS THAT WORK FOR 1500 MHz GO DOWN TO 30 MHz
      PL=PL-(FREQ/250)*ALOG10(1500/FREQ)
      IF(RANGE.GT.25) PL=PL-0.18*ALOG10(1500/FREQ)*(RANGE-25)
C COMPUTE FREE SPACE PATH LOSS IN DBI
      PL2=36.6+20*ALOG10(FREQ)+20*ALOG10(RANGE)
      RETURN
      END
```

5.2 Anderson 2D Model

The Anderson 2D model is a comprehensive point-to-point radio propagation model for predicting field strength and path loss in the frequency range of 30 MHz to 60 GHz. This model draws upon techniques which have been successfully used for many years, such as those described in NBS Technical Note 101 [1], and improves upon them by making use of widely available terrain elevation and local land use (ground cover) databases. As described in Section 5.5, this model can also be extended to provide for the first time 3D modeling of reflections from terrain features which are not along the great circle path between the transmitter and the receiver. Such reflections result in multipath and time-dispersed signal energy at the receiver. Such an extension is important for prediction the performance of certain digital systems where time-dispersed reflections are a primarily cause of irreducible data errors due to inter-symbol interference (ISI).

The model specification is divided into several sections which describe its various components. Section 5.2.1 is a basic model outline which describes how the components fit together. Sections 5.2.2 and 5.2.3 define the model for the line-of-sight (LOS) and non-line-of-sight (NLOS), respectively. Section 5.4 discusses the local clutter attenuation and the uses of the land use/ land cover database to incorporate this attenuation. The Anderson 2D model also includes a troposcatter mode for long-range over the horizon path loss prediction, and atmospheric absorption loss which is relevant at frequencies above 10 GHz. For the systems for which this Report is intended, the troposcatter mode and atmospheric absorption loss are not applicable and will not be described here.

The level of detail in this specification is in keeping with scientific standards. Equations and specific information are provided such that knowledgeable researchers in the field can replicate the model in computer code and reproduce the model results. However, no computer code or pseudo code is provided here since approaches to implementation can vary widely.

The Anderson 2D model is supported for administrative purposes in spectrum management and regulation. As such it has been designed to take into account the more important elements of propagation prediction while still remaining simple enough so that computer implementation is straightforward and the model can be broadly applied.

An important objective in designing the model described in this document was to make it simple and thereby accessible. In keeping with this objective, however, it is recognized that the defined model is not the most complete possible solution to predicting electromagnetic (EM) fields in a complex propagation environment. Other approaches such as the Integral Equation (IE) and Parabolic Equation (PE) methods could potentially provide more accurate full-wave solutions but with attendant limitations and a substantial increase in complexity. The model defined in this document relies on the geometric optic (ray-tracing) approach which basically deals with the transport of EM energy from location to another. It is an easy technique to visualize, and conceptually it is readily adapted to the 3D extension for predicting multipath and time-dispersion. Attempting to use IE or PE techniques in a full 3D mode for this purpose would be a daunting computational task, even on the largest computers.

5.2.1 Propagation Model Outline

For the purposes of this report, the Anderson 2D model has three basic elements which affect the predicted field strength at the receiver as follows:

1) Line-of-Sight (LOS) mode using basic two-ray theory with constraints

2) Non-line-of-sight (NLOS) mode using multiple wedge diffraction

3) Local clutter attenuation (see Section 5.4 of this report)

The LOS and NLOS modes are mutually exclusive - a given path between a transmitter and receiver is either LOS or not. The local clutter loss is an integral part of this model which is necessary to achieve correct signal level predictions in suburban, urban, and forested areas. It is describe separately in Section 5.4 of this Report.

The fundamental decision as to whether a path is LOS is based on the path geometry. It is described in the next section which defines the LOS mode for this model.

5.2.2 Line-of-Sight (LOS) Mode

The determination of whether a path between transmitter and receiver is LOS is done by comparing the depression angle of the path between the transmitter and receiver with the depression angle to each terrain elevation point along the path. The depression angle from transmitter to receiver is computed using an equation of the form of (6.15) in [15]:

$$\theta_{t-r} = \frac{h_r - h_t}{d_r} - \frac{d_r}{2a} \qquad \text{[Eq. 14]}$$

where:

θ_{t-r} is the depression angle relative to horizontal from the transmitter to the receiver in radians

h_t is the elevation of the transmit antenna center of radiation above mean sea level in meters

h_r is the elevation of the receive antenna center of radiation above mean sea level in meters

d_r is the great circle distance from the transmitter to the receiver in meters

a is the effective earth radius in meters taking into account the atmospheric refractivity

The atmospheric refractivity is usually called the K factor. A typically value of K is 1.333, and using an actually earth radius of 6340 kilometers, a would equal 8451 kilometers, or 8,451,000 meters.

Using an equation of the same form, the depression angle from the transmitter to any terrain elevation point can be found as:

$$\theta_{t-p} = \frac{h_p - h_t}{d_p} - \frac{d_p}{2a} \qquad \text{[Eq. 15]}$$

where:

θ_{t-p} is the depression angle relative to horizontal for the ray between the transmitter and the point on the terrain profile

h_p is the elevation of the terrain point above mean sea level in meters

d_p is the great circle path distance from the transmitter to the point on the terrain path in meters

h_t and a are defined above

The variable θ_{t-p} is calculated at every point along the path between the transmitter and the receiver and compared to θ_{t-r}. If the condition $\theta_{t-p} > \theta_{t-r}$ is true at any point, then the path is considered NLOS and the model formulations in Section 5.2.3 are used. If $\theta_{t-p} \leq \theta_{t-r}$ is true at every point, then the transmitter-receiver path is LOS and the formulations in this section apply.

For LOS paths the field strength at the receiver is calculated as the vector combination of a directly received ray and a single reflected ray. This calculation is presented in Section 5.2.2.1. If the geometry is such that a terrain elevation point along the path between the transmitter and receiver extends into the 0.6 Fresnel zone, then an additional loss ranging from 0 to 6 dB is included for partial Fresnel zone obstruction. This is discussed in Section 5.2.2.2.

5.2.2.1 Two-Ray Field Strength at the Receiver Using a Single Ground Reflection

For an LOS path, the field at the receiver consists of the directly received ray from the transmitter and number of other rays received from a variety of reflecting and scattering sources. For low antenna heights (on either the transmit or receive end of the path) the field at the receiver is dominated by the direct ray and a single reflected ray which intersects the ground near the transmitter. The *height-gain function* in which at field at the antenna increases as the height of the antenna above ground increases is a direct result of the direct and ground refection rays vectorially adding so that the magnitude of the resultant manifests this effect. The height-gain function is modeled here by considering the actual ground reflected ray and direct ray in vector addition. The magnitude of the direct ray is given by:

$$E_r = \frac{1}{d_r}\sqrt{\frac{P_T G_T \eta}{4\pi}} \qquad \text{[Eq. 16]}$$

where E_r is the field strength at the receive point, P_T is the transmitter power delivered to the terminals of the transmit antenna, G_T is the transmit antenna gain in the direction of the receiver point (or ray departure direction), η is the plane wave free space impedance (377 ohms), and d_r is the path distance from the transmitter to the receive point in kilometers.

Written in dB terms, this reduces to the familiar:

$$E_r = 76.92 - 20.0\log(d_r) + P_T \quad \text{dB}\mu\text{V / m} \qquad \text{[Eq. 17]}$$

In [Eq. 17], P_T is effective radiated power (ERP_d) in dBW. The magnitude and phase of the ground-reflected ray is found by first calculating the complex reflection coefficient as follows:

$$R = R_s g \qquad \text{[Eq. 18]}$$

where R_s is the smooth surface reflection coefficient and g is the surface roughness attenuation factor (a scalar quantity).

For parallel and perpendicular polarizations, respectively, the smooth surface reflection coefficients are:

$$R_{s\|} = \frac{\sin\gamma_0 - \sqrt{\varepsilon - \cos^2\gamma_0}}{\sin\gamma_0 + \sqrt{\varepsilon - \cos^2\gamma_0}} \quad \text{parallel polarization} \qquad \text{[Eq. 19]}$$

$$R_{s\perp} = \frac{\varepsilon\sin\gamma_0 - \sqrt{\varepsilon - \cos^2\gamma_0}}{\varepsilon\sin\gamma_0 + \sqrt{\varepsilon - \cos^2\gamma_0}} \quad \text{perpendicular polarization} \qquad \text{[Eq. 20]}$$

where γ_0 is the angle of incidence and ε is the complex permittivity given by:

$$\varepsilon = \varepsilon_1 - j60\sigma_1\lambda \qquad \text{[Eq. 21]}$$

where ε_1 is the relative dielectric constant of the reflecting surface, σ_1 is the conductivity of the reflecting surface in Siemens/m, and λ is the (free space) wavelength of the incident radiation. For the case of a ground reflection, vertical polarization is parallel polarization and horizontal polarization is perpendicular polarization.

For the model defined here, it will be assumed that the local surface roughness is 0 (smooth surface) so that the term g in [Eq. 18] is one. Also, values of σ_1 = 0.008 Siemens/meter and that ε_i = 15 are commonly used for ground constants.